

## Grade 8 Pythagorean Theorem (Relationship)

8.SS.1	
Develop and apply the Pythagorean theorem to solve problems.	<ol style="list-style-type: none"><li>1. Model and explain the Pythagorean theorem concretely, pictorially, or by using technology.</li><li>2. Explain, using examples, that the Pythagorean theorem applies only to right triangles.</li><li>3. Determine whether or not a triangle is a right triangle by applying the Pythagorean theorem.</li><li>4. Determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem.</li><li>5. Solve a problem that involves Pythagorean triples (e.g., 3, 4, 5 or 5, 12, 13).</li></ol>

### Clarification of the outcome:

- ◆ This outcome concerns understanding and being able to solve basic triangle problems involving the Pythagorean theorem (for students: Pythagorean relationship):

$$\textit{side1}^2 + \textit{side2}^2 = \textit{hypotenuse}^2$$

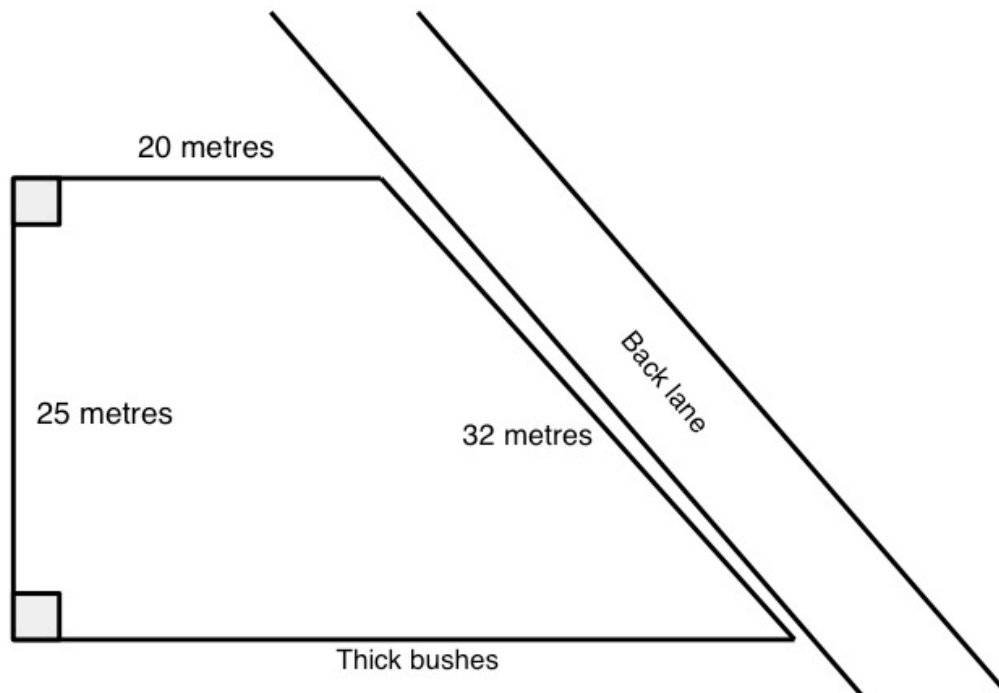
### Required close-to-at-hand prior knowledge:

- ❖ Can identify and draw right triangles.
- ❖ Understand angle measurement.
- ❖ Understand squaring and square root
- ❖ Can solve basic equations of the form:  $A + ? = C$  and  $? + B = C$
- ❖ Able to determine the area of rectangles and triangles (grades 6 and 7 outcomes)

## SET SCENE stage

Provide students with the following problem.

My backyard is weird shape. Right now the dandelions are winning the battle between grass and weeds. My parents want to plow up the backyard and put new sod on it. To do that, they need to know the area of the backyard. They can't figure it out so they asked me to take the problem to school for my classmates to figure out. Here is a diagram of my backyard. I can't measure the length of the south side because thick bushes get in the way.



### The problem task to present to students:

Ask an actual student to present the problem and then ask his/her classmates to try to solve it in whatever way they want. That could include placing a transparent cm grid over the diagram, approximating the length of the south side, etc.

### Comments:

The purpose of the task is present a situation that will lead to a reason for learning the Pythagorean relationship.

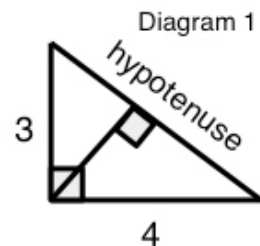
## DEVELOP stage

### Activity 1: Revisits SET SCENE

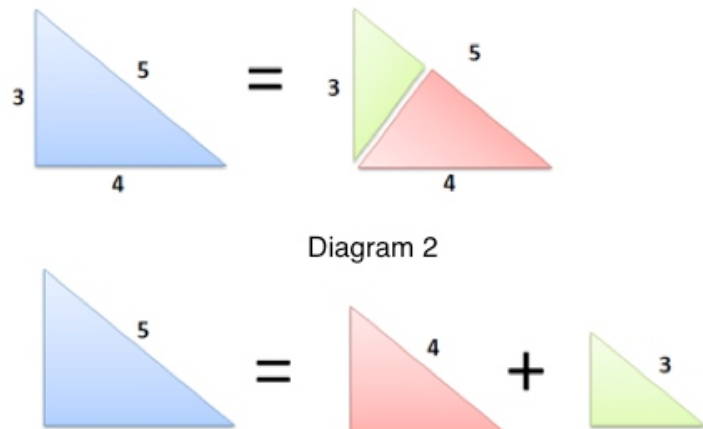
Ask selected students to discuss how they tried to solve the SET SCENE problem and what solution they obtained. Do not indicate correct/not correct. Just acknowledge the approaches. If no student used this approach, point out that the area problem involves the area of a rectangle add the area of a right triangle. The trouble is that the base of the right triangle is missing. Tell students they will return to this after they learned more about right triangles.

### Activity 2: Addresses achievement indicators 1 and 2 (loosely), and “prepares the garden”.

- ◆ Provide 1 cm grid paper. Ask students to draw a right triangle having side lengths of 3 and 4. Ask students to measure the length of the long side (name it - hypotenuse, the side across from the right triangle). Students should obtain a length of 5 cm for the hypotenuse.
- ◆ Ask students to draw a line from the right angle vertex (corner) to the hypotenuse so that the line is perpendicular to the hypotenuse (see diagram 1).



- ◆ Ask students to colour the two smaller triangles formed using different colours (see diagram 2). Ask students to cut along the perpendicular line they drew, thus separating the big triangle into two smaller triangles. Ask students to indicate the length of the hypotenuse for each smaller triangle (see diagram 2). Ensure they can do that.



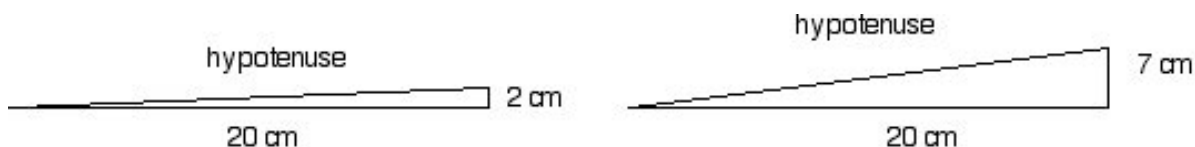
- ◆ Ask students whether the area of the triangle having hypotenuse 4 add the area of the triangle having hypotenuse 3 equals the area of the original triangle (having hypotenuse 5). Ensure they realize that the area of the big triangle = area of small triangle #1 + area of small triangle #2. Mention that they should keep this thinking in mind if they want to discover a secret about right triangles.

#### Note:

The purposes of activity 2 are: (1) to ensure understanding of critical parts of a right triangle and (2) to introduce addition of areas (this will facilitate students discovering the Pythagorean relationship).

**Activity 3: Addresses achievement indicators 2 and 4 (loosely) and “prepares the garden”.**

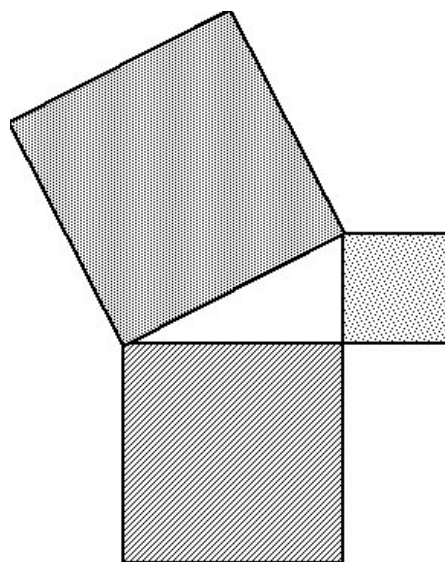
- ◆ Using 1 cm grid paper, ask students to draw a series of right triangles for which the height is respectively 2 cm, 7 cm, 12 cm, 27 cm, and 32 cm (see diagram). The base for each triangle is always 20 cm. For each triangle, ask them to estimate the length of the hypotenuse and then measure it. Discuss the results.



- ◆ Ask students what a possible relationship might be that would allow them to figure out the length of the hypotenuse, knowing the base and height (side 1 and side 2) of the right triangle. Discuss and test some of their ideas about what the relationship might be. Students should know realize that the relationship may not be as simple as (for example) add base and height (the two sides) to figure out the hypotenuse.

**Activity 4: Addresses achievement indicators and 1 and 2.**

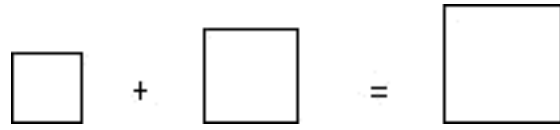
- ◆ Using graphics software or 1 cm grid paper, have students draw a 3-4-5 right triangle and construct squares on each of the three sides (see diagram). Have students fit the two smaller squares into the large square by subdividing the smaller squares and then cutting and pasting.
- ◆ Ask students if being able to fit the two smaller squares into the large one reveals the secret about a right triangle. Remind students about the result from activity 2. [It is unlikely that a student will discover the theorem at this point. The following illustrates a way of guiding students to discover the secret. SEE NEXT PAGE.]



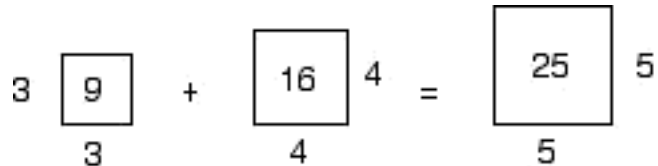
**Note:**

If using 1 cm grid paper, it is useful to have students make two copies of the situation. One copy serves as a visual reference; the other copy is for cutting and pasting.

- Ask students what it means in terms of area if the two smaller squares fit exactly inside the larger square. Lead them to realize that it means that the area of square #1 + the area of square #2 = the area of square #3 (the one formed from the hypotenuse).



- Ask students how to calculate the area of each square. Enter the results in the picture form shown here.



- Ask if anyone sees a relationship between the three sides of the right triangle. Guide them by writing  $3 \times 3 + 4 \times 4 = 5 \times 5$  and remind them about a short way to indicate multiplying by the same number. Ensure students gain a preliminary sense of  $3^2 + 4^2 = 5^2$ .
- ◆ Repeat the above approach (draw triangle, fit squares, area relationship, etc.) for a 5-12-13 right triangle. Ensure that students realize that the squares on the two sides fit into the square on the hypotenuse and then "discover" the Pythagorean relationship (see below).

$$\text{(hypotenuse)}^2 = \text{(side 1)}^2 + \text{(side 2)}^2$$

**Activity 5: Addresses achievement indicators 2 and 5.**

- ◆ Provide students with diagrams showing 3-4-5 and 5-12-13 right triangles, where one side is missing (see example). Ask students to determine the missing side. [Students should simply recall 3-4-5 and 5-12-13 as being the sides of right triangles.]
- ◆ Tell students that 3-4-5 and 5-12-13 are Pythagorean triplets. Ask them to check if (6, 8, 10); (7, 24, 25); and (9, 40, 41) are also Pythagorean triplets.
- ◆ Discuss that the triplets only apply to right triangles.



**Activity 6: Addresses achievement indicator 1 and practice.**

- ◆ Have students use graphics software or 1 cm grid paper to draw right triangles and measure the lengths of the three sides. Students enter the length data into a spreadsheet and confirm the relationship between the lengths of the three sides by squaring the shorter sides (base and height), adding their squares, and then comparing the sum of those squares to the square of the hypotenuse. Ask for and discuss results.

**Note:**

**Drawing right triangles on 1 cm grid paper is one way to reduce error in measurement. The lengths of the two sides of a right triangle can easily and accurately be determined from the 1 cm grid lines. The student only needs to use a 30 cm ruler to measure the length of the hypotenuse.**

**Activity 7: Addresses achievement indicators 2 and 3.**

Ask students to draw triangles other than right triangles. Have them measure side lengths and check to see if the Pythagorean relationship works for non-right triangles. Ask for and discuss results. Ensure they realize that the Pythagorean relationship only works for a right triangle.

**Activity 8: Addresses achievement indicators 2 and 4, and closure on SET SCENE.**

- ◆ Revisit the SET SCENE problem. Ask students to redraw the diagram so that a right triangle is clearly present. Ask students how that might help with figuring out the area of the backyard. Ensure they realize the area of the backyard is the area of the rectangle add the area of the triangle.
- ◆ Ask students what information is needed to determine the area of the right triangle. Ensure they realize that one side length is not known (the base of the triangle). Ask students to figure out the length of the missing side. Ensure they use Pythagoras and are able to do the following:

$$25^2 + base^2 = 32^2$$

$$base^2 = 32^2 - 25^2$$

$$base^2 = 1024 - 625$$

$$base^2 = 399$$

$$base = \sqrt{399} = 19.97$$

- ◆ Discuss that 19.97 is almost 20, and that using 20 is okay when laying sod because there is always waste. Therefore it is wise to order more than the area requires. Ensure students can complete the area calculation by doing:  $25 \times 20$  (rectangle area) +  $1/2 \times 25 \times 20$  (triangle area) =  $500 + 250 = 750$  sq. m.

**Activity 9: Addresses achievement indicator 5 & SHIFT to flat notation**

- ◆ Provide students with word problems that require the Pythagorean relationship. Restrict the problems to one application of the relationship. Ask for and discuss solutions. Below is a sample problem. [Solution: *The widest part of the desk (the diagonal) cannot fit through the doorway because the diagonal is greater than 1.4 m.*]

A doorway is 1.4 m wide. A square desk has side length of 1 m. Is it possible for the widest part of the desk to fit through the doorway? Explain.

- ◆ Provide students with websites that require the Pythagorean relationship to solve problems. Ask for and discuss solutions. Here is a sample website.

[Practice with Pythagorean Theorem](#)

**Activity 10: Assessment of teaching.**

- Provide students with a simple problem that involves determining the length of the hypotenuse. Have students solve the problem by drawing a labelled diagram and using the Pythagorean relationship.
- Provide students with a simple problem that involves determining the length of one of the sides (base or height). Have students solve the problem by drawing a labelled diagram and using the Pythagorean relationship.

*If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).*

*An example of a partial well-designed worksheet follows.*

*The worksheet contains a sampling of question types. More questions of each type are needed.*

*The MAINTAIN stage follows the sample worksheet.*

### Question 1.

Determine if each statement is true or false.

- a)  $4^2 + 5^2 = 6^2$
- b)  $9^2 + 12^2 = 15^2$

### Question 2.

Determine the value of H.

- a)  $10^2 + 24^2 = H^2$
- b)  $5^2 + 10^2 = H^2$

### Question 3.

Determine the value of A.

- a)  $A^2 + 16^2 = 20^2$
- b)  $4^2 + A^2 = 10^2$

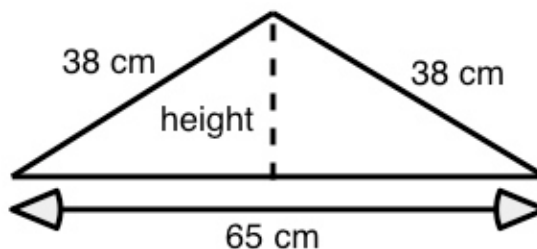
### Question 4.

- a) The numbers 3, 4, 5 are known as Pythagorean triplets because they are whole numbers that form the sides of right triangles. Determine whether the following sets of numbers are Pythagorean triplets. See if you can figure out a way of determining that without having to use the Pythagorean relationship.
  - i) 30, 40, 50
  - ii) 300, 400, 500
  - iii) 5, 7, 9
- b) The numbers 5, 12, 13 are also Pythagorean triplets. Create two more triplets based on 5, 12, 13 by using the way figured out in part a). Use the Pythagorean relationship to confirm that the three sides are sides of a right triangle.

### Question 5.

Solve the following problem:

*Bob is building a dollhouse for his sister's birthday. He is ready to build the roof. Find the height of the roof to the nearest tenth of a cm if the house is 65 cm across and each slanted roof side is 38 cm long.*





## MAINTAIN stage

### Mini-task example

Every so often:

- Present an equation involving the Pythagorean relationship. Ask students to solve the equation. Ask them to draw the shape that applies to the equation. Here is an example of an equation.

$$7^2 + X^2 = 20^2$$

### Rich-task example

A tangram set consists of 7 pieces: 1 square, 2 large triangles, 1 medium triangles, 2 small triangles, and 1 parallelogram.

Designate the side length of the square of as 1 unit. Ask students to determine the lengths of all sides of all seven tangram pieces.



### Comments

This is a rich-task because it is a complex problem involving the Pythagorean relationship and the examination of a variety and number of shapes.